For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Ms. Pickett is building a picket fence around her house. She wants to build the fence so that all points on the fence are exactly 6 feet away from the sides of her house. If her house is an equilateral triangle with side length of 6 feet, what is the total length of the fence, in feet?
(A) 18 (B) $18+3\sqrt{3}$ (C) $36\sqrt{3}$ (D) $18+6\sqrt{3}$ (E) NOTA

2. Sri really likes ice cream. He also really likes spheres. If he has a perfectly full tub of ice cream that is in the shape of a cylinder with radius 3 cm and height 4 cm, how many perfectly full ice cream spheres of radius 2 cm can he make? Assume Sri is magical and none of the ice cream melts.
(A) $\frac{27}{8}$ (B) $\frac{14}{4}$ (C) 4 (D) 5 (E) NOTA

3. Rayyan's favorite line is defined by $4x+3y=6$. Jason's favorite line is perpendicular to Rayyan's favorite line. What is the slope of a line perpendicular to Jason's favorite line?
(A) $\frac{3}{4}$ (B) $-\frac{4}{3}$ (C) $\frac{4}{3}$ (D) $-\frac{3}{4}$ (E) NOTA

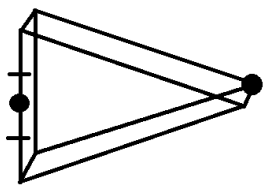
4. As the president of Rickards MAO, Kyle got a shirt with an odd symbol inscribed on it. The symbol can be defined as a circle of radius 12 units interconnected with a decagon. The circle is centered at $(0,0)$ and the decagon has points at $(2,4)$ and $(12,10)$. The area outside of the circle but inside of the decagon is equal to the area outside of the decagon but inside of the circle. Kyle, being a nerd, calculates the area of the decagon. What is this area?
(A) $80\sqrt{7}$ (B) 140π (C) $80\sqrt{7}-144\pi$ (D) 144π (E) NOTA

5. Find the inverse of the converse of the contrapositive of: "If my granny is 414 years old, then she is wise."
(A) If my granny is not wise, then she is not 414 years old. (B) If my granny is wise, then she is 414 years old.
(C) If my granny is not 414 years old, then she is not wise. (D) If my granny is 414 years old, then she is wise.
(E) NOTA

6. Alex finds that mustard complements his chicken well. Find the measure of the angle whose complement is 30 less than one-half the measure of its supplement.
(A) 60 (B) 120 (C) 330 (D) 30 (E) NOTA

7. Consider the parabola $y=2x^2-13x-357$. Find the shortest distance between the positive x-intercept of this parabola and the line $y=-\frac{4}{3}x-\frac{4}{3}$.
(A) 17 (B) $-\frac{4}{3}$ (C) $\frac{72}{5}$ (D) $\frac{72\sqrt{17}}{17}$ (E) NOTA

8. Nihar has a very long tongue. (I'm not joking, have you seen that thing?!) Anyway, his tongue is in the shape of a triangular prism, with height 1 inch and with isosceles triangular bases with side lengths 5, 5, and 3 inches. What is the length from the upper tip of his tongue to the center of the lower back edge, the base of the isosceles triangles (from one dot to the other), of his tongue? All answers are expressed in inches.



(A) $\frac{\sqrt{95}}{4}$ (B) $\frac{\sqrt{95}}{2}$ (C) $\frac{\sqrt{91}}{4}$ (D) $\frac{\sqrt{91}}{2}$ (E) NOTA

9. Dylan likes to draw triangles. He draws one with vertices at $(34, 12)$, $(-16, -27)$, and $(26, -5)$. He wants to balance this triangle on the tip of his pencil. Assuming Dylan meticulously sharpens his pencil into a single point, what is the point on which this triangle will balance?
(A) $(15, -7)$ (B) $(22, -10)$ (C) $(\frac{44}{3}, -\frac{20}{3})$ (D) $(\frac{35}{3}, -5$ (E) NOTA

10. Rohan likes potatoes. He brings home 100 pounds of them. The potatoes are 99 percent water. Being a fickle person, he dehydrates the potatoes overnight until they are 98 percent water. How much do the potatoes weigh now, in pounds?
(A) 99 (B) 98 (C) 50 (D) 25 (E) NOTA

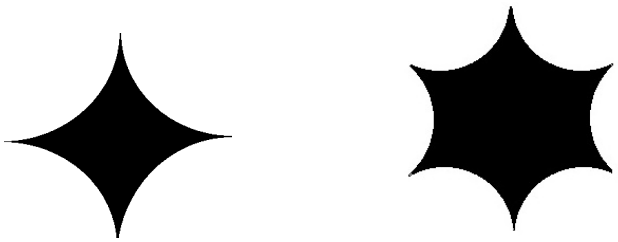
11. In triangle ABC, AD is the angle bisector of BAC, AB is 12, AC is 16, and DC is half of the square of BD. What is the length of BD?
(A) $\frac{3}{2}$ (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) 4 (E) NOTA

12. Two circles intersect and have a common chord of length x. The distance between the two circles is y and the radius of one of the circles is z. If $\frac{x}{2} < z < y$, what is the radius of the second circle, in terms of x, y, and z?
(A) $\sqrt{y^2 + z^2 - y\sqrt{4z^2 - x^2}}$ (B) $\sqrt{y^2 + z^2 - y\sqrt{z^2 - \frac{x^2}{4}}}$ (C) $\sqrt{x^2 + y^2 + z^2 - y\sqrt{z^2 - x^2}}$
(D) $\sqrt{y^2 - z^2 + y\sqrt{4z^2 - x^2}}$ (E) NOTA

13. What is the area of an equilateral triangle inscribed in a circle inscribed in a regular hexagon of side length 4?
(A) $4\sqrt{3}$ (B) $9\sqrt{3}$ (C) $8\sqrt{3}$ (D) $6\sqrt{3}$ (E) NOTA

14. Joshua is baking a cake, but as a beginner baker, he has no idea how much frosting to put on the outside of it. If the cake is in the shape of a right regular hexagonal frustum with slant height 2, and the side lengths of each base are 4 and 6, what is the total surface area needed to be covered in frosting, assuming all sides of the caked are frosted?
(A) $78\sqrt{3}$ (B) $108\sqrt{3}$ (C) $30 + 78\sqrt{3}$ (D) $60 + 78\sqrt{3}$ (E) NOTA

15. Deekshita and Diyah are stargazing. Deekshita sees a regular four pointed star with perimeter 4π, as shown, while Diyah sees a regular six pointed star with perimeter 8π, as shown. What is the absolute value of the difference between the areas of the two stars?



(A) $16 + 4\pi - 24\sqrt{3}$ (B) $24\sqrt{3} - 4\pi - 16$ (C) $24\sqrt{3} - 4\pi$ (D) $16 + 8\pi$ (E) NOTA

16. What is the sum of the degree measures of all of the interior angles in a polygon with 120 sides, in degrees?
(A) 360 (B) 2124 (C) 21240 (D) 21600 (E) NOTA

17. Tanusri has a hollow charm in the shape of a right cone with radius 4 and height 8. Tanvi has a spherical charm that she finds can be perfectly placed (inscribed) in Tanusri's charm. What is the radius of Tanvi's charm?
(A) $2\sqrt{5} - 2$ (B) $\frac{8}{3}$ (C) $4 - \sqrt{3}$ (D) $\frac{\sqrt{5}+1}{8}$ (E) NOTA

18. A kite with side lengths of 5 and 12 is inscribed in a circle. What is the radius of the circle?
(A) 7 (B) 13 (C) 17 (D) $\frac{17}{2}$ (E) NOTA

For problems 19-21, consider a triangle ABC with side lengths $AB = 12$, $BC = 35$, and $AC = 37$. A median BD is drawn to side AC, with D on AC.

19. What is the secant of angle A?
(A) $\frac{37}{12}$ (B) $\frac{37}{35}$ (C) $\frac{12}{37}$ (D) $\frac{35}{37}$ (E) NOTA

20. Which angle has a measure equal to $\arcsin\left(\frac{12}{37}\right)$?
(A) $\angle BCA$ (B) $\angle CAB$ (C) $\angle BDC$ (D) $\angle ADB$ (E) NOTA

21. What is the length of the median BD?
(A) $\frac{37}{2}$ (B) $\frac{47}{2}$ (C) $\frac{49}{2}$ (D) $\frac{37\sqrt{2}}{2}$ (E) NOTA

22. Find the slope of the Euler line of the triangle with vertices $(5, 5)$, $(8, 5)$, and $(5, 9)$.
(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) 1 (D) $-\frac{3}{4}$ (E) NOTA

23. Vishnav and Akash are rich, and want to take a taxi to school together. They must leave in between 8:30 AM and 8:50 AM. However, neither likes waiting, and they are both only willing to wait 5 minutes before leaving in a personal taxi alone. Assuming both randomly arrive within the 20 minute period, what is the probability that they are able to meet and leave in a taxi together, instead of wasting gas and driving in two separate taxis?
(A) $\frac{7}{16}$ (B) $\frac{9}{16}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$ (E) NOTA

24. What is one of the intersection points between the parabola $y = 2x^2 + 6x + 7$ and the line $y = -x + 4$?
(A) $\left(-3, \frac{9}{2}\right)$ (B) $(-3, 7)$ (C) $\left(\frac{1}{2}, \frac{9}{2}\right)$ (D) $\left(\frac{1}{2}, 7\right)$ (E) NOTA

25. Two congruent circles intersect so that each passes through the center of the other. Given that each of the circles has a radius of r, what is the area of the overlapping region, in terms of r?
(A) $\frac{r^2\sqrt{3}}{2}$ (B) $\frac{4\pi r^2 - 3r^2\sqrt{3}}{6}$ (C) $\frac{4\pi r^2 - \sqrt{3}}{6}$ (D) $\frac{4\pi r^2 - 6r^2\sqrt{3} + 4r}{6}$ (E) NOTA

26. What is the radius of a circle with equation $x^2 + y^2 - 10x - 12y + 43 = 0$?
(A) 43 (B) 5 (C) 18 (D) 6 (E) NOTA

27. The surface area of one dodecahedron is 9. A similar dodecahedron has a surface area of 16. What is the ratio of the volume of the first dodecahedron to the second?
(A) $\frac{3}{4}$ (B) $\frac{9}{16}$ (C) $\frac{7}{16}$ (D) $\frac{1}{4}$ (E) NOTA

28. Cherry is quizzing Karthik, and Karthik is trying not to act like a fool. Cherry asks Karthik, "What is the sum of the number of sides and number of diagonals in a regular triskaidecagon?" Assuming Karthik proves that he is not a fool, what is his answer?
(A) 435 (B) 360 (C) 253 (D) 78 (E) NOTA

29. A ninja named Ninju is standing 5 miles north of a perfectly straight river. He wants to get to a dojo 10 miles east and 5 miles north of his current location. However, he is very thirsty, and wants to stop by the river first for a drink of water. What is the shortest distance Ninju can travel if he wants to stop by the river before heading to the dojo?
(A) $10 + 5\sqrt{3}$ (B) $5 + 10\sqrt{2}$ (C) 20 (D) $5\sqrt{13}$ (E) NOTA

30. Consider a cyclic quadrilateral with side lengths 5, 6, 9, and 10. What is the area of this quadrilateral?
(A) $30\sqrt{3}$ (B) $90\sqrt{5}$ (C) $27\sqrt{5}$ (D) $45\sqrt{10}$ (E) NOTA